Diffusion processes for operational management of a wind farm

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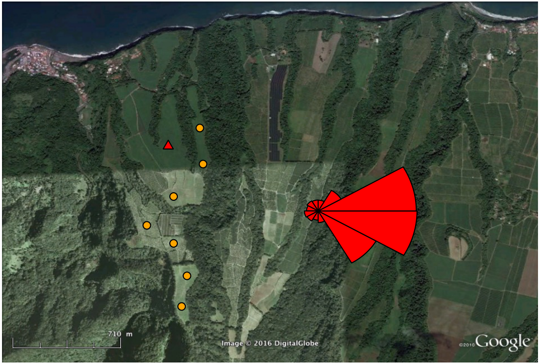


Uncertainties:

- wind speed,
- vertical extrapolation,
- horizontal extrapolation,
- power function,
- long-term extrapolation,
- measures,
- cut-out, unavailability.

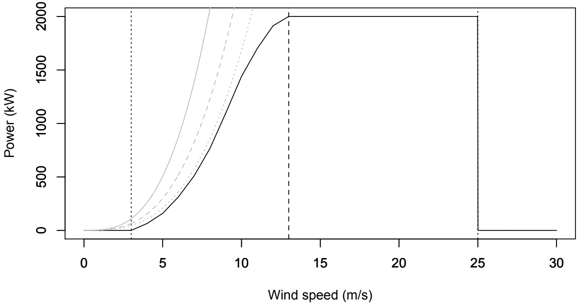
Management:

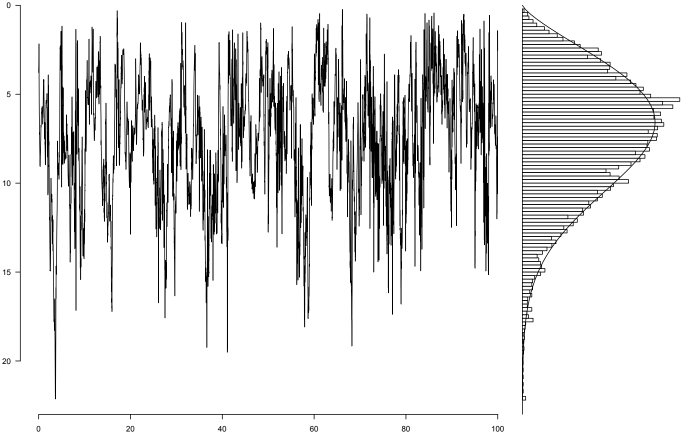
- safe regulation (s, m),
- storage, trading (h),
- maintenance (d).

Power of the wind stream $P_w(v) = \frac{1}{2}\rho S v^3$.

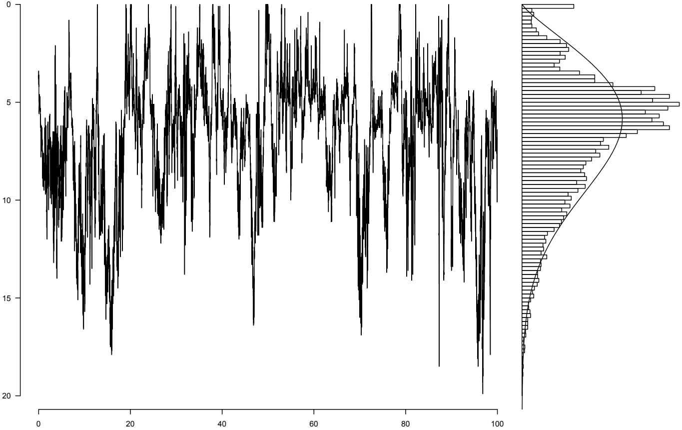
Betz' limit of wind turbine power $P_{max}(v) = \frac{16}{27} P_w(v)$.

This power is transformed and we consider the power (transfer) function $P(v)$ (cannot reach in practice 70% of the Betz' limit).

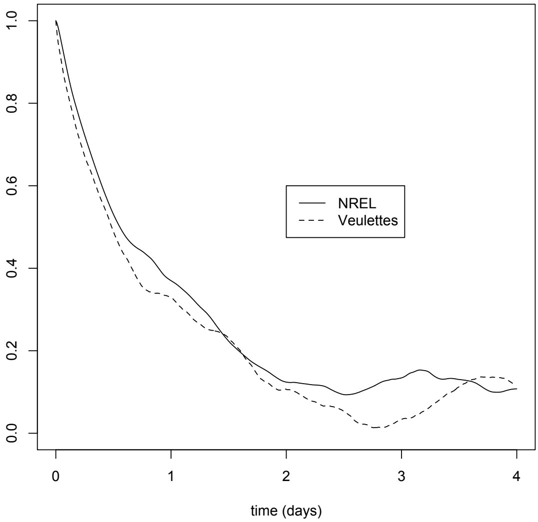




Simulated wind speed dataset of the NREL.



Real wind speed dataset in Veulettes-sur-Mer, France.


1.0
0.8
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0.4
0.2
0.0
0
1
2
3
4
time (days)
NREL
Veulettes

Lots of studies of dynamical models for modeling and forecasting:

– statistical models (time series, Markov models, neural networks, . . .) for seconds, minutes, hours;

– meteorological models for days and weeks.

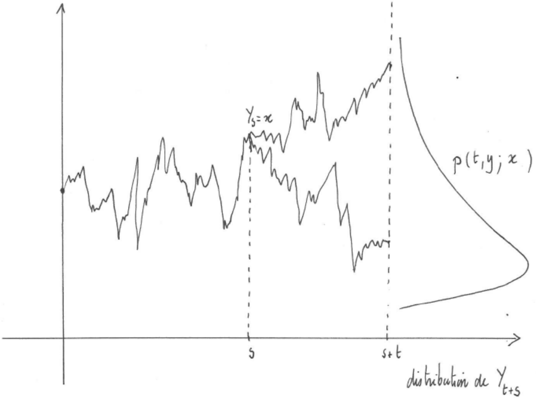
We are studying two particular diffusion process models and consider their evaluation for modeling and short-term forecasting. They are entries for stochastic control problems for storage and trading.

We consider homogeneous diffusion processes that are Markov processes for which the transition probability density functions $p(t, y\,; x, \vartheta)$ satisfy the Fokker-Planck-Kolmogorov equation

$$\frac{\partial}{\partial t} p = -\frac{\partial}{\partial y}(v_0(y, \vartheta) p) + \frac{1}{2} \frac{\partial^2}{\partial y^2}(v_1(y, \vartheta)^2 p), \quad y \in \mathbb{R}, \quad t > 0,$$

with initial condition $p(0, y\,;\, x, \vartheta) = \delta(y - x)$.

We have evaluated the Cox-Ingersoll-Ross (CIR) model (B. and Bensoussan 2016) and the 3-parameter marginal Weibull model (B. and Bensoussan, preprint). The transition probability density functions are in closed form for the CIR model (Feller, 1951) but not for the marginal Weibull diffusion model.



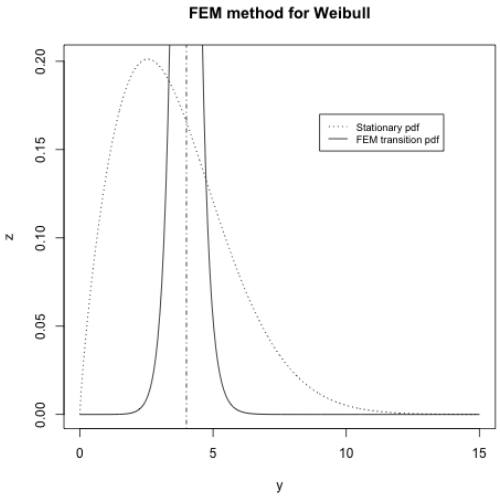
$$E_x(Y_{s+t}) \underset{t \to 0}{\simeq} x + V_0(x, \theta)\, t$$

$$Var_x(Y_{s+t}) \underset{t \to 0}{\simeq} V_1^2(x, \theta) \cdot t$$

CIR

$$V_0(x, \theta) = \theta_1(\theta_2 - x)$$

$$V_1(x, \theta) = \theta_3 \sqrt{x}$$


FEM method for Weibull
Stationary pdf
FEM transition pdf
z
y
0.00
0.05
0.10
0.15
0.20
0
5
10
15

Construction of the 3-parameter marginal Weibull diffusion model is inspired from (Bibby, Skovgaard et Sorensen, 2003). We compute v_1 such that the stationary distribution is Weibull.

Using the F-P-K equation, the stationary distribution f satisfies

$$-\frac{\partial}{\partial y}(v_0(y,\vartheta)f(y)) + \frac{1}{2}\frac{\partial^2}{\partial y^2}(v_1(y,\vartheta)^2 f(y)) = 0, \quad y \in \mathbb{R}.$$

and by integration

$$-v_0(y,\vartheta)f(y) + \frac{1}{2}\frac{\partial}{\partial y}(v_1(y,\vartheta)^2 f(y)) = 0, \quad y \in \mathbb{R}.$$

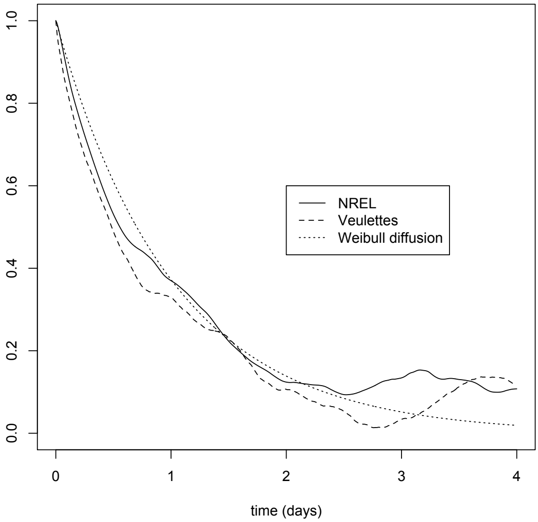
Given the stationary distribution f and v_0, we can find v_1^2.

Fixing the linear drift $v_0(y, \vartheta) = \vartheta_1(\vartheta_2 - y)$ and integrating the previous equation leads to

$$\int_{\mathbb{R}} v_0(y, \vartheta) f(y) dy = 0 \quad \Longrightarrow \quad \vartheta_2 = \int_{\mathbb{R}} y\, f(y)\, dy.$$

It can be shown generally that for the mean-reverting diffusion process $\left(Y_t^{y_0}, t \geq 0\right)$

$$\lim_{s \to \infty} \frac{\mathbf{E}\left(\left(Y_s^{y_0} - \mathbf{E}\, Y_s^{y_0}\right)\left(Y_{s+t}^{y_0} - \mathbf{E}\, Y_{s+t}^{y_0}\right)\right)}{\mathbf{var}\, Y_s^{y_0}} = \exp\left(-\vartheta_1 t\right).$$


1.0
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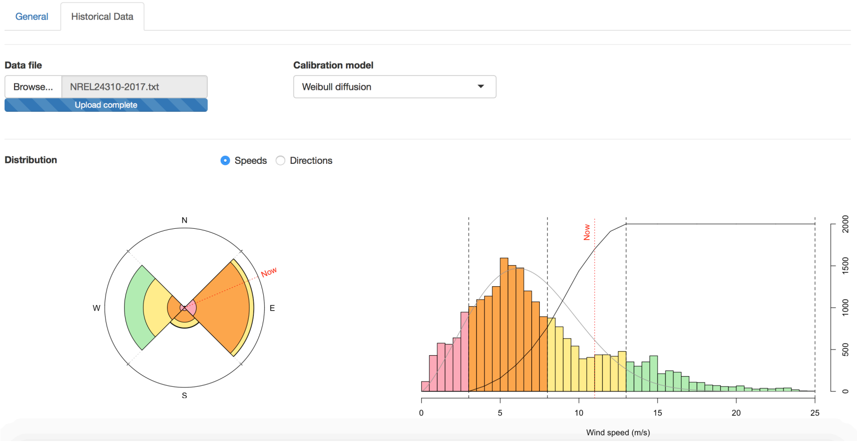
time (days)
NREL
Veulettes
Weibull diffusion


General
Historical Data
Data file
Browse...
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Upload complete
Calibration model
Weibull diffusion
Distribution
Speeds
Directions
N
W
E
S
Now
Now
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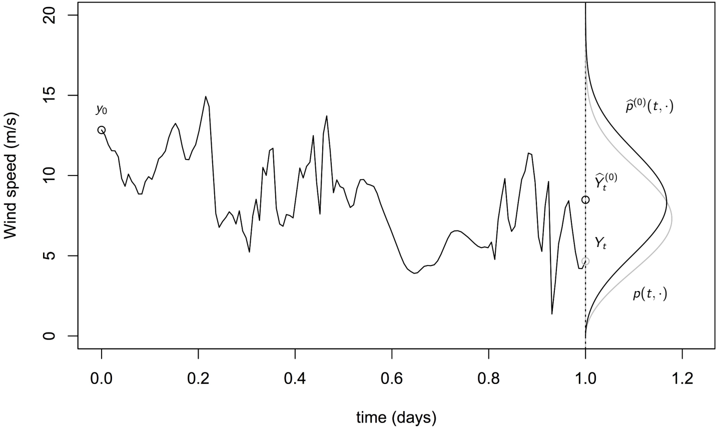
5
10
15
20
25
Wind speed (m/s)
0
500
1000
1500
2000


Wind speed (m/s)
0
5
10
15
20
y_0
$\widehat{Y}_t^{(0)}$
Y_t
$\widehat{p}^{(0)}(t,\cdot)$
$p(t,\cdot)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
time (days)

Homogeneous diffusion processes $\left(Y_t^{y_0}, t \geq 0\right)$ admit a natural point forecast

$$\widehat{Y}_t^{(0)} = \mathbf{E}_\vartheta\left(Y_t^{y_0}\right) = \int_{\mathbb{R}} y\, p(t, y\,;\, y_0, \vartheta) dy$$

and a probabilistic forecast with their transition densities

$$\widehat{p}^{(0)}(t, \cdot) = p(t, \cdot\,;\, y_0, \vartheta).$$

Cox-Ingersoll-Ross diffusion and marginal Weibull diffusion processes with linear drift term $v_0(y, \vartheta) = \vartheta_1(\vartheta_2 - y)$ have an explicit point forecast

$$\mathbf{E}_\vartheta\left(Y_t^{y_0}\right) = \vartheta_2 + (y_0 - \vartheta_2)\exp\left(-\vartheta_1 t\right).$$

For a well-specified model setting ($Y_t^{obs} = Y_t^{y_0}$), we have

$$MSE(t) = \mathbf{E}\left((Y_t^{y_0})^2\right) - \left(\mathbf{E}Y_t^{y_0}\right)^2 = u(t, y_0) - \left(\mathbf{E}Y_t^{y_0}\right)^2$$

with $u(t,x)$ solving the Feynman-Kac pde, *i.e.*

$$\frac{\partial u}{\partial t} = v_0(x,\vartheta)\frac{\partial u}{\partial x} + \frac{v_1^2(x,\vartheta)}{2}\frac{\partial^2 u}{\partial x^2}$$

with

$$u(0,x) = x^2.$$

General | Historical Data

Now		Speed	*11 m/s*	Direction	*23 °*	Production	*1.6 MW*
Model	Weibull diffusion		*scale=7.8 shape=2.3 correlation parameter=1.4*				
Forecast	1 h	Speed	*10.8 m/s*	Direction	*23 °*	Production	*1.6 MW*
Reliability	Expected time to failure		*--- h*	Expected exit time	*--- h*		

It is possible to show van Trees inequality in several statistical experiments, namely, for any sequence of estimators $(T_n, n \geq 1)$,

$$\liminf_{C\to\infty} \liminf_{n\to\infty} \sup_{|\vartheta-\vartheta_0|<C\varphi_n(\vartheta_0)} \mathbf{E}_\vartheta \left(\ell\left(\varphi_n(\vartheta_0)^{-1}(T_n-\vartheta)\right)\right) \geq \mathbf{E}\left(\ell\left(I(\vartheta_0)^{-\frac{1}{2}}\xi\right)\right)$$

where ξ is a standard Gaussian random variable.

We are looking for an asymptotically efficient sequence of estimators that reaches the lower bound.

Let $\Theta \subset \mathbb{R}^d$. A family of measures $\{P_\theta^n, \theta \in \Theta\}$ is called locally asymptotically normal (LAN) at $\theta_0 \in \Theta$ if there exist **nondegenerate** $d \times d$ matrices $\varphi_n(\theta_0)$ and $I(\theta_0)$ such that for any $u \in \mathbb{R}^d$, the likelihood ratio

$$Z_n(u) = \frac{\mathrm{d}P^n_{\theta_0+\varphi_n(\theta_0)u}}{\mathrm{d}P^n_{\theta_0}}$$

admits the representation

$$Z_n(u) = \exp\left(\langle u, \zeta_n(\theta_0)\rangle - \frac{1}{2}\langle I(\theta_0)u, u\rangle + r_n(\theta_0, u)\right), \quad (1)$$

where

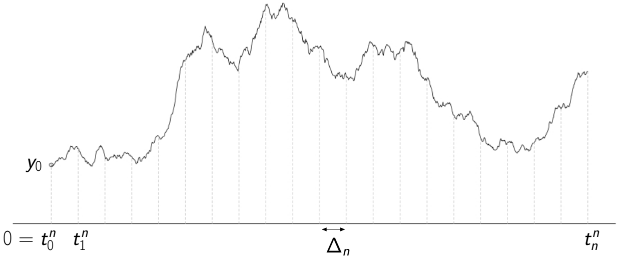
$$\zeta_n(\theta_0) \to \mathcal{N}(0, I(\theta_0)), \quad r_n(\theta_0, u) \to 0 \quad (2)$$

in law under $P^n_{\theta_0}$.

LAN property of the likelihoods has been established for several statistical experiments:

1. sample of i.i.d. r.v. (second Le Cam's Lemma);
2. sample of independent but inhomogeneous r.v. with the Lindeberg condition;
3. sequence of an ergodic Markov chain;
4. strictly elliptic and ergodic diffusions;
5. diffusions with observational noise;
6. Levy processes;
7. fractional Gaussian noise;
8. and others ...

Let $(Y_t, t \geq 0)$ be the solution of a (fractional) SDE whose law depends on the unknown parameter ϑ.



Our aim is to give asymptotical properties of estimators of ϑ given the observation of the path on a discrete grid $0 < t_1^n < \ldots < t_n^n$, as $n \to \infty$. Asymptotic properties depend on the convergence scheme.

Large sample. Here $\Delta_n = \Delta > 0$ is fixed and, under proper assumptions (smoothness, ergodicity, uniform ellipticity), the LAN property of the likelihoods is satisfied (Roussas 72) with rate $\varphi(n) = \frac{1}{\sqrt{n}}$ and the Fisher information matrix is equal to

$$\mathcal{I}(\Delta, \vartheta)_{i,j} = \int_{\mathbb{R}} \int_{\mathbb{R}} \frac{\partial}{\partial \vartheta_i} \log p \frac{\partial}{\partial \vartheta_j} \log p \cdot p \, dy \mu_\vartheta(dx) \qquad (3)$$

where μ_ϑ is the invariant measure of the diffusion process. Consequently, the lower bound for the variance of the estimators can be derived precisely for any sequence of estimators $(\vartheta_n, n \geq 1)$,

$$\lim_{C \to \infty} \liminf_{n \to \infty} \sup_{|\vartheta - \vartheta_0| < \frac{C}{\sqrt{n}}} \mathbf{E}_\vartheta \ell \left(\sqrt{n} \left(\widetilde{\vartheta}_n - \vartheta \right) \right) \geq \mathbf{E}_{\vartheta_0} \ell \left(\mathcal{I}(\vartheta_0)^{-1} \xi \right)$$

with $\xi \sim \mathcal{N}(0, I)$, and ℓ is a polynomial cost function.

Mixed scheme: Here $n\Delta_n \to \infty$, $\Delta_n \to 0$ and the LAN property of the likelihoods have been established (Gobet, 2002) under proper conditions (smoothness, ergodicity, uniform ellipticity) with different rates for ϑ_1 (drift parameter) and ϑ_2 (diffusion coefficient parameter). Namely $\varphi(n)_{1,1} = \frac{1}{\sqrt{n\Delta_n}}$ and $\varphi(n)_{2,2} = \frac{1}{\sqrt{n}}$, respectively, and the Fisher information matrix is given by

$$\mathcal{I}(\vartheta)_{i,j} = \int_{\mathbb{R}} \frac{\partial}{\partial \vartheta_{1,i}} v_0(y,\vartheta_1) \frac{\partial}{\partial \vartheta_{1,j}} v_0(y,\vartheta_1) \cdot v_1(y,\vartheta_2)^{-2} \mu(dy)$$

and

$$\mathcal{I}(\vartheta)_{q+i,q+j} = 2 \int_{\mathbb{R}} \frac{\partial}{\partial \vartheta_{2,i}} v_1(y,\vartheta_2) \frac{\partial}{\partial \vartheta_{2,j}} v_1(y,\vartheta_2) \cdot v_1(y,\vartheta_2)^{-2} \mu(dy).$$

The YUIMA Project is mainly developed by statisticians who actively publish in the field of inference for stochastic differential equations.

The YUIMA Project Core Team with write access to the source code, currently consists of:

- Alexandre Brouste (Le Mans)
- Masaaki Fukasawa (Osaka)
- Hideitsu Hino (Waseda U.)
- Stefano M. Iacus (Milan)
- Kengo Kamatani (Tokyo)
- Hiroki Masuda (Kyushu U.)
- Yasutaka Shimizu (Osaka)
- Masayuki Uchida (Osaka)
- Nakahiro Yoshida (Tokyo)

1– A. Bensoussan and A. Brouste (2016) Cox-Ingersoll-Ross model for wind speed modeling and forecasting, Wind Energy, 19(7), 1355-1365.

2*– A. Bensoussan and A. Brouste *Marginal Weibull diffusion model for wind speed modeling and short-term forecasting*, preprint.

3*– A. Brouste and M. Fukasawa *Local asymptotic normality property for fractional Gaussian noise under high-frequency observations*, preprint.

4– A. Brouste, M. Fukasawa, H. Hino, S. Iacus, K. Kamatani Y. Koike, H. Masuda, R. Nomura, Y. Shimuzu, M. Uchida and N. Yoshida (2014) The YUIMA Project : a Computational Framework for Simulation and Inference of Stochastic Differential Equations, Journal of Statistical Software, 57(4), 1-51.